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Ultra-large distance modification of gravity from Lorentz symmetry breaking at the Planck scale

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Abstract: We present an extension of the Randall-Sundrum model in which, due to spontaneous Lorentz symmetry breaking, graviton mixes with bulk vector fields and becomes quasilocalized. The masses of KK modes comprising the four-dimensional graviton are naturally exponentially small. This allows to push the Lorentz breaking scale to as high as a few tenth of the Planck mass. The model does not contain ghosts or tachyons and does not exhibit the van Dam-Veltman-Zakharov discontinuity. The gravitational attraction between static point masses becomes gradually weaker with increasing of separation and gets replaced by repulsion (antigravity) at exponentially large distances.

Keywords: Field Theories in Higher Dimensions, Large Extra Dimensions, p-branes.

JHEP09(2005)082

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Contents

1. Introduction **1**

2. The model **3**

3. Tensor modes **7**

4. Vector modes **11**

5. Scalar sector **12**

6. Discussion **18**

A. Junction conditions in the scalar sector **20**

B. Absence of ghosts above a non-trivial background **21**

1. Introduction

The discovery of cosmological acceleration [1] indicates the necessity for modification of the standard cosmological paradigm. One way to explain the accelerated expansion of the Universe is to attribute it to the effect of some unconventional matter such as the cosmological constant or quintessence. In this approach one encounters, however, the difficulty of explaining the small value of the associated energy scale in comparison with other fundamental scales, such as the Planck mass or electroweak symmetry breaking scale. It is reasonable to consider an alternative possibility that the cosmological constant is exactly zero, but its effect is mimicked by modifications of gravitational laws at distance scale of the order of the present cosmological horizon.

A natural framework for modifying gravity at ultra-large distances is provided by models with infinite extra spatial dimensions. A number of brane-world models have been proposed [2, 3] where the four-dimensional graviton is a superposition of bulk KK modes, and thus, is not localized on the brane. Rather, it is quasilocalized, having a non-zero width Γ (and, possibly, mass m) with respect to decay into extra dimensions. In these models gravity is four-dimensional at distances smaller than $r_c \sim \min\{m^{-1}, \Gamma^{-1}\}$ and becomes multi-dimensional at larger scales. The attractive feature of the models with extra-dimensions is that the tiny values of m, Γ are generated without strong fine-tuning of parameters.

However, Lorentz invariant extra dimensional models with infrared modifications of gravity face the problems [4 – 6] akin to those of four-dimensional massive gravity. Either these theories contain ghosts, fields with the wrong sign of kinetic term, or the propagator of graviton exhibits the van Dam-Veltman-Zakharov (vDVZ) discontinuity [7] originating

JHEP09(2005)082

from a scalar degree of freedom that does not decouple in the massless limit. At the classical level the discontinuity might be cured by nonlinear effects [8, 9], but at the quantum level the presence of the additional scalar leads to strong coupling [10, 5, 6] at the energy scale at most of order $(m^2 M_{Pl})^{1/3}$. If m is comparable to the present Hubble parameter of the Universe, the theory is in strong coupling regime, and thus looses its predictivity, at distances smaller than 1000 km.

There are indications that these difficulties might be resolved in the Dvali-Gabadadze-Porrati (DGP) [3] model. In particular, it was argued that the scale of strong coupling can be pushed to higher energies in nontrivial classical backgrounds, provided the structure of counterterms in the model is of a special form [9]. However, it is still unclear whether the counterterms in the DGP model actually meet the requirements of ref. [9]. Probably, the most disappointing fact about the DGP model is that the branch of the cosmological solutions in this theory, which exhibits cosmic acceleration without introduction of the cosmological constant, is plagued by ghost-like instabilities [5, 9].

In this paper we propose a brane-world model with infrared modification of gravity and violation of 4-dimensional Lorentz-invariance. In our model graviton becomes quasilocalized due to mixing with vector fields which freely propagate in the bulk. This kind of mixing becomes possible once the Lorentz symmetry is spontaneously broken by condensates of the vector fields.

Our approach builds up on the ideas put forward recently in the four-dimensional framework. It was suggested that the problems of massive gravity could be resolved in models incorporating violation of Lorentz invariance [11 – 13]. Models with explicit Lorentz violation in the gravitational sector were considered in refs. [11 – 14] and it was shown that they are equivalent to a class of models with gravity coupled to scalar fields with unusual kinetic terms. In the latter formulation the Lorentz symmetry is broken spontaneously by coordinate dependent scalar condensates. Because of nonlinear structure of the scalar kinetic term in this kind of models, they possess a cutoff scale $\Lambda_{\rm cutoff} = (m M_{Pl})^{1/2}$, and can be considered only as low energy effective σ-models. If the graviton mass m is of the order of the present Hubble parameter, then $\Lambda_{\rm cutoff} \approx (0.01{\rm mm})^{-1}$, which is phenomenologically acceptable, but still unnaturally small in comparison with other fundamental scales. It is unclear whether these σ-models can be extended to complete models making sense above the scale $\Lambda_{\rm cutoff}$.

As a scenario for such a complete model one could envisage a gravitational analog of the Higgs mechanism triggered by spontaneous Lorentz symmetry breaking. In refs. [15 – 19] it was proposed to use coordinate independent vector (in general, tensor) condensates for this purpose. However, in four-dimensional theories, graviton does not acquire mass within this approach: no gap in the dispersion relation of graviton appears [20, 21]. Moreover, the effect of vector condensates on cosmology amounts to nothing but the change of the gravitational constant in the cosmological equations [19, 21], and no cosmological acceleration is produced.

The brane-world approach opens up a new way to circumvent the problems encountered in four dimensions. In this paper we make the first step along this line: in section 2 we present a model in which graviton becomes quasilocalized due to spontaneous Lorentz

JHEP09(2005)082

symmetry breaking induced by non-zero VEVs of bulk vector fields, and analyze in the subsequent sections the spectrum of linear perturbations about four-dimensionally flat background. In section 3 we analyze the tensor perturbations and find that the characteristic mass m_c of KK modes comprising four-dimensional graviton is naturally exponentially small, which allows to take the Lorentz-breaking scale as high as a few tenth of the Planck mass. In section 4 and section 5 we study vector and scalar perturbations and show that the model contains neither tachyons, nor ghosts, and does not exhibit the vDVZ discontinuity. We calculate long distance modification of the Newton law and find quite unexpectedly that attraction of point masses changes to repulsion at distances of order $1/m_c$. Thus, the model provides a realization of antigravity at ultra-large distances. This looks promising in view of the possibility to obtain cosmological acceleration at late times. On the other hand, modification of gravitational field of static sources in the model faces severe phenomenological constraints. We discuss possible ways to get around these constraints in the concluding section 6.

A number of important issues are left beyond the scope of the present paper. These are, e.g., cosmology of the proposed model and structure of quantum corrections. We leave them for future investigation.

2. The model

We consider a five-dimensional setup with positive tension brane and negative cosmological constant in the bulk — the Randall-Sundrum (RS) model [22]. We add to this model three bulk vector fields A^a_M, $a = 1, 2, 3$, with quartic potential localized on the brane. The action is taken in the following form,

$$S = \int d^5x\sqrt{g}\Big(-\frac{R}{16\pi G_5} - \Lambda - \frac{1}{4}F^a_{MN}F^{a\,MN}\Big) + \tag{2.1}$$
$$+ \int d^4x\sqrt{-\bar{g}}\left(-\sigma - \frac{\varkappa_1^2}{2}\big(\bar{g}^{\mu\nu}A^a_\mu A^b_\nu + v^2\delta^{ab}\big)^2 - \frac{\varkappa_2^2}{2}\left(\bar{g}^{\mu\nu}A^a_\mu A^b_\nu - \frac{1}{3}\delta^{ab}\bar{g}^{\mu\nu}A^c_\mu A^c_\nu\right)^2\right),$$

where $\bar{g}_{\mu\nu}$ is the induced metric on the brane, and

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M\,.$$

The capital Latin indices M, N take values $0, 1, 2, 3, 5$, while the Greek indices $\mu, \nu, \ldots$ run from 0 to 3. The action (2.1) is invariant under global SO(3) symmetry with vector fields A^a_M belonging to fundamental representation. It is straightforward to check that the potential in the second line of eq. (2.1) is generic quartic potential invariant under this group. Note that the second term in the potential depends only on the traceless part of the matrix $\bar{g}^{\mu\nu}A^a_\mu A^b_\nu$. The parameters v^2, $\varkappa_1^2$, $\varkappa_2^2$ are assumed to be positive. In addition to the global SO(3) symmetry, the bulk part of the action is invariant under U(1) × U(1) × U(1) gauge transformations,

$$A^a_M \mapsto A^a_M + \partial_M\alpha^a\,.$$

This abelian gauge symmetry is broken explicitly at the brane.

JHEP09(2005)082

The model has a static solution with AdS metric in the bulk and constant values of the vector fields,

$$ds^2 = -dz^2 + \mathrm{e}^{-2k|z|}\eta_{\mu\nu}dx^\mu dx^\nu\,, \tag{2.2a}$$

$$A_5^a = A_0^a = 0\,, \qquad A_i^a = v\delta_i^a\,, \qquad i = 1,2,3\,, \tag{2.2b}$$

where $k = \sqrt{-4\pi G_5\Lambda/3}$, and the usual fine-tuning $\sigma = -\Lambda/k$ is assumed; the signature of the Minkowski metric $\eta_{\mu\nu}$ is $(+,-,-,-)$. The VEVs (2.2b) of the vector fields break SO(3)×Lorentz symmetry down to SO(3) of spatial rotations accompanied by simultaneous rotations in the internal space. The similar pattern of Lorentz symmetry breaking was considered in the four-dimensional context in refs. [23, 21]. As we will see, this spontaneous symmetry breaking provides mixing between the graviton zero mode, present in the pure RS case, and the continuum spectrum of KK modes of vector fields. This mixing results in quasilocalization of the graviton.

Let us study the linearized perturbations above the background (2.2). Imposing the gauge conditions $g_{5\mu} = 0$, $g_{55} = -1$ on the metric, one writes the following decomposition,

$$ds^2 = -dz^2 + (\mathrm{e}^{-2k|z|}\eta_{\mu\nu} + h_{\mu\nu}(x,z))dx^\mu dx^\nu\,,$$
$$A_M^a = v\delta_M^a + a_M^a(x,z)\,.$$

The analysis is simplified by the fact that the energy-momentum tensor of the vector fields in the bulk vanishes to the linear order in perturbations, allowing for fixing the transverse traceless gauge in the bulk [22, 24],

$$h_\mu^\mu = h^\mu_{\nu,\mu} = 0\,. \tag{2.3}$$

Here and throughout the paper indices $\mu, \nu, \ldots$ are raised (lowered) using the metric $\eta^{\mu\nu}$ ($\eta_{\mu\nu}$), and comma denotes derivative. In this gauge the bulk equations for the metric take a fairly simple form,

$$\frac{1}{2}h''_{\mu\nu} - 2k^2 h_{\mu\nu} - \frac{1}{2[u(z)]^2}h_{\mu\nu,}{}^{,\lambda}_{\lambda} = 0\,, \tag{2.4}$$

where $u(z) = \mathrm{e}^{-k|z|}$, and prime denotes derivative with respect to z. As to the vector fields, the bulk equations read

$$-a_\nu^{a\prime\prime} + 2ka_\nu^{a\prime} + \frac{1}{u^2}a^{a\,,\mu}_{\nu\,,\mu} = 0\,. \tag{2.5}$$

In deriving this equation we made use of the $[\mathrm{U}(1)]^3$ gauge invariance of the bulk action to impose the conditions

$$a_5^a = 0\,, \qquad a^{a\,\mu}{}_{,\mu} = 0 \tag{2.6}$$

on the vector fields in the bulk. Imposing the two conditions (2.6) simultaneously is possible, as they are compatible on-shell. The coordinate frame where the conditions (2.3) and (2.6) are satisfied will be referred to below as the bulk frame.

The boundary conditions on the brane are most easily formulated in the Gauss normal (GN) reference frame. In this frame the brane is fixed at $\bar{z} = 0$ (the quantities with the bar refer to the GN frame), and we assume Z_2 symmetry across the brane. Then, for general energy-momentum tensor $T_{\mu\nu}$ on the brane, the boundary conditions for the metrics at

JHEP09(2005)082

$\bar{z} = +0$ read [24],

$$\bar{h}'_{\mu\nu} + 2k\bar{h}_{\mu\nu} = 8\pi G_5 \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T^{\lambda}_{\lambda} \right). \tag{2.7}$$

The linearized energy-momentum tensor of the vector fields comes from the Lorentz violating term in the quadratic action arising from the brane potential in eq. (2.1). To the quadratic order, the latter term has the form,

$$S^{(2)}_{LV} = -\int d^4x \left[\frac{\varkappa_1^2 v^4}{2} \left(\bar{h}_{ab} + \frac{1}{v}(\bar{a}^a_b + \bar{a}^b_a) \right)^2 + \right.$$
$$\left. + \frac{\varkappa_2^2 v^4}{2} \left(\bar{h}_{ab} + \frac{1}{v}(\bar{a}^a_b + \bar{a}^b_a) - \frac{1}{3}\delta_{ab} \left(\bar{h}_{cc} + \frac{2}{v}\bar{a}^c_c \right) \right)^2 \right]. \tag{2.8}$$

From this expression one obtains,

$$T^V_{00} = T^V_{0a} = 0\,, \tag{2.9a}$$
$$T^V_{ab} = 2\varkappa^2 v^4 \left(\bar{h}_{ab} + \frac{1}{v}(\bar{a}^a_b + \bar{a}^b_a) \right) - \frac{2\varkappa_2^2 v^4}{3}\delta_{ab} \left(\bar{h}_{kk} + \frac{2}{v}\bar{a}^k_k \right), \tag{2.9b}$$

where we introduced $\varkappa^2 = \varkappa_1^2 + \varkappa_2^2$. Note that the energy-momentum tensor (2.9) of the vector fields violates the weak energy condition. We will show that this property does not lead to instabilities. At the same time it is crucial for antigravity emerging at ultra-large distances.

For the vector fields, the Z_2 symmetry and continuity demand that the 5th component vanishes on the brane, $\bar{a}^a_5(\bar{z} = 0) = 0$. The sources for the other components can be read off from (2.8), resulting in the following boundary conditions at $\bar{z} = +0$,

$$\bar{a}^{a\prime}_0 = 0\,, \tag{2.10a}$$
$$\bar{a}^{a\prime}_i = \varkappa^2 v^3 \left(\bar{h}_{ai} + \frac{1}{v}(\bar{a}^a_i + \bar{a}^i_a) \right) - \frac{\varkappa_2^2 v^3}{3}\delta_{ai} \left(\bar{h}_{kk} + \frac{2}{v}\bar{a}^k_k \right). \tag{2.10b}$$

The metrics in the bulk frame and GN frame are related by a gauge transformation. The form of the latter is restricted by the condition $\bar{h}_{55} = \bar{h}_{5\mu} = 0$. One obtains

$$\bar{h}_{\mu\nu} = h_{\mu\nu} + u^2(\varepsilon_{\mu,\nu} + \varepsilon_{\nu,\mu}) + \frac{1}{k}\varepsilon_{,\mu\nu} - 2ku^2\eta_{\mu\nu}\varepsilon\,, \tag{2.11}$$

where the functions ε, ε_μ depend only on x. This transformation corresponds to the following change of coordinates,

$$z = \bar{z} + \varepsilon\,, \qquad x^\mu = \bar{x}^\mu + \frac{1}{2ku^2}\varepsilon^{,\mu} + \varepsilon^\mu\,.$$

Note that in the bulk frame the brane is displaced from the origin along the z-direction, its position being described by the brane bending ε. The relation between the components of the vector fields in the two frames has the form

$$\bar{a}^a_5 = a^a_5 - \frac{v}{u^2}\varepsilon_{,a} + \partial_5\alpha^a\,,$$
$$\bar{a}^a_\mu = a^a_\mu - \frac{v}{2ku^2}\varepsilon_{,a\mu} + v\varepsilon^a_{,\mu} + \alpha^a_{,\mu}\,,$$

JHEP09(2005)082

where we allowed for the possibility of a $[\mathrm{U}(1)]^3$ gauge transformation parametrized by the functions $\alpha^a(z,x)$. Imposing the gauge $\bar{a}^a_5 = 0$, which is consistent with the boundary conditions on the brane, we obtain,

$$\bar{a}^a_\mu = a^a_\mu + v\varepsilon^a_{,\mu} + \beta^a_{,\mu}\,, \tag{2.12}$$

where the functions β^a depend only on x. The set of equations (2.7), (2.9)–(2.11) and (2.12) results in the following boundary conditions,

$$h'_{00} + 2kh_{00} + 2\varepsilon_{,00} = \frac{\lambda_1}{3}\left(h_{aa} + \frac{2}{v}a^a_a + \frac{1}{k}\varepsilon_{,aa} + 6k\varepsilon + \frac{2}{v}\beta^a{}_{,a}\right)\,, \tag{2.13a}$$

$$h'_{0i} + 2kh_{0i} + 2\varepsilon_{,0i} = 0\,, \tag{2.13b}$$

$$h'_{ij} + 2kh_{ij} + 2\varepsilon_{,ij} = \lambda\Big\{h_{ij} + \frac{1}{v}(a^i_j + a^j_i) + \frac{1}{k}\varepsilon_{,ij} + + \frac{1}{v}(\beta^i_{,j} + \beta^j_{,i}) - \\ - \frac{1}{3}\delta_{ij}\left(h_{kk} + \frac{2}{v}a^k_k + \frac{1}{k}\varepsilon_{,kk} + \frac{2}{v}\beta^k{}_{,k}\right)\Big\}\,, \tag{2.13c}$$

$$a^{a\prime}_0 = 0\,, \tag{2.13d}$$

$$a^{a\prime}_i = \varkappa^2 v^3\Big\{h_{ai} + \frac{1}{v}(a^a_i + a^i_a) + \frac{1}{k}\varepsilon_{,ai} + \frac{1}{v}(\beta^a_{,i} + \beta^i_{,a})\Big\} + 2\varkappa_1^2 v^3 k\delta_{ai}\varepsilon - \\ - \frac{\varkappa_2^2 v^3}{3}\delta_{ai}\left(h_{kk} + \frac{2}{v}a^k_k + \frac{1}{k}\varepsilon_{,kk} + \frac{2}{v}\beta^k_{,k}\right); \tag{2.13e}$$

here we introduced the notations

$$\lambda_1 = 16\pi G_5 \varkappa_1^2 v^4\,, \qquad \lambda = 16\pi G_5 \varkappa^2 v^4\,.$$

In what follows it is convenient to work in the 4-dimensional Fourier representation,

$$h_{\mu\nu}, a^a_\mu \propto \mathrm{e}^{-ip_0x_0 + i\mathbf{px}}\,,$$

and decompose the perturbations into scalar, vector and tensor modes with respect to rotations around the three-momentum $\mathbf{p}$. To this end one introduces a three-dimensional orthogonal basis consisting of $\mathbf{p}$ and two unit vectors, $\mathbf{e}^{(\alpha)}$, $\alpha = 1, 2$. The latter are used to construct a pair of transverse traceless symmetric tensors,

$$d^{(1)}_{ij} = \frac{1}{\sqrt{2}}\left(e^{(1)}_i e^{(2)}_j + e^{(1)}_j e^{(2)}_i\right), \qquad d^{(2)}_{ij} = \frac{1}{\sqrt{2}}\left(e^{(1)}_i e^{(1)}_j - e^{(2)}_i e^{(2)}_j\right),$$

and a transverse antisymmetric tensor,

$$f_{ij} = \frac{1}{\sqrt{2}}\left(e^{(1)}_i e^{(2)}_j - e^{(1)}_j e^{(2)}_i\right).$$

The tensorial decomposition reads,

$$h_{00} = \phi_1\,, \tag{2.14a}$$

$$h_{0i} = p_i\phi_2 + e^{(\alpha)}_i\psi_{1\alpha}\,, \tag{2.14b}$$

JHEP09(2005)082

$$h_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2}\right)\phi_3 + p_i p_j \phi_4 + \left(p_i e_j^{(\alpha)} + p_j e_i^{(\alpha)}\right)\psi_{2\alpha} + d_{ij}^{(\alpha)}\chi_{1\alpha}\,, \tag{2.14c}$$

$$a_0^a = p_a \phi_5 + e_a^{(\alpha)}\psi_{3\alpha}\,, \tag{2.14d}$$

$$a_i^a = \left(\delta_{ia} - \frac{p_i p_a}{\mathbf{p}^2}\right)\phi_6 + p_i p_a \phi_7 + \left(p_i e_a^{(\alpha)} + p_a e_i^{(\alpha)}\right)\psi_{4\alpha} + \left(p_i e_a^{(\alpha)} - p_a e_i^{(\alpha)}\right)\psi_{5\alpha} + \\ + d_{ia}^{(\alpha)}\chi_{2\alpha} + f_{ia}\xi\,, \tag{2.14e}$$

$$\beta^a = p_a \phi_8 + e_a^{(\alpha)}\psi_{6\alpha}\,, \tag{2.14f}$$

with $\mathbf{p} = (p_1, p_2, p_3)$. There are two fields χ in the symmetric tensor sector; one field ξ in the sector of antisymmetric tensors; six fields ψ in the vector sector, and eight fields ϕ in the scalar sector. Note that all fields in (2.14), except for ϕ_8 and ψ_6, are functions of the fifth coordinate z.

3. Tensor modes

Let us study the spectrum of tensor perturbations. To begin with, one notices that the antisymmetric field ξ completely decouples. It satisfies eq. (2.5) with the free boundary condition $\xi'(z=0) = 0$, and forms a continuum spectrum of completely delocalized modes similar to the modes of a free vector field in RS background [25]. We do not consider these modes below.

For the symmetric tensors one obtains from eqs. (2.4), (2.5) the following equations in the bulk,

$$\frac{1}{2}\chi_1'' - 2k^2\chi_1 + \frac{p^2}{2u^2}\chi_1 = 0\,, \tag{3.1a}$$

$$-\chi_2'' + 2k\chi_2' - \frac{p^2}{u^2}\chi_2 = 0\,, \qquad z > 0\,, \tag{3.1b}$$

where $p^2 \equiv p_0^2 - \mathbf{p}^2$, and polarization index α is omitted to simplify notations. The boundary conditions are read off from eqs. (2.13c), (2.13e):

$$\chi_1' + 2k\chi_1 = \lambda\left(\chi_1 + \frac{2}{v}\chi_2\right), \tag{3.2a}$$

$$\chi_2' = \varkappa^2 v^3\left(\chi_1 + \frac{2}{v}\chi_2\right), \qquad z = +0\,. \tag{3.2b}$$

The system (3.1), (3.2) can be rewritten in the form of an eigenvalue problem,

$$-u^2(\chi_1'' - 4k^2\chi_1) - \left[(4k - 2\lambda)\chi_1 - 2\mu\hat{\chi}_2\right]\delta(z) = m^2\chi_1\,, \tag{3.3a}$$

$$-u^2(\hat{\chi}_2'' - 2k\hat{\chi}_2') + \left[2\mu\chi_1 + 2\nu\hat{\chi}_2\right]\delta(z) = m^2\hat{\chi}_2\,, \tag{3.3b}$$

where we denoted $m^2 = p^2$, $\nu = 2\varkappa^2 v^2$, $\mu = \sqrt{\lambda\nu}$ and introduced $\hat{\chi}_2 = \sqrt{32\pi G_5}\chi_2$ to make the operator hermitean with the scalar product

$$(\eta, \chi) = \int_{-\infty}^{+\infty} dz \left(\frac{\eta_1^*\chi_1}{u^2} + \hat{\eta}_2^*\hat{\chi}_2\right). \tag{3.4}$$

JHEP09(2005)082

First, let us make sure that there are no tachyonic modes. The operator (3.3) is the sum of the diagonal operator appearing in the pure RS case and a positive semi-definite operator

$$\Delta = \begin{pmatrix} 2\lambda & 2\mu \\ 2\mu & 2\nu \end{pmatrix} \delta(z)\,.$$

Thus, the absence of tachyons in our case is ensured by their absence in the RS model.

Next, there is no normalizable zero mode. Indeed, if $m^2 = 0$, then $\chi_1 = A_1 \mathrm{e}^{-2k|z|}$, $\hat{\chi}_2 = A_2 = const$ (other solutions to eqs. (3.1) are exponentially growing). From eqs. (3.2) one obtains that $2\lambda A_1 + 2\mu A_2 = 0$. Thus, $A_2 \neq 0$ and the zero mode is not normalizable with the scalar product (3.4).

We now turn to the modes with $m^2 \geq 0$. These modes belong to continuum spectrum. Introducing the conformal coordinate $\zeta = \mathrm{e}^{k|z|}/k$ one writes the general solution of the bulk equations (3.1) as a combination of Bessel functions,

$$\chi_1 = A_1 J_2(m\zeta) + B_1 N_2(m\zeta)\,, \tag{3.5a}$$

$$\hat{\chi}_2 = m\zeta\big[A_2 J_1(m\zeta) + B_2 N_1(m\zeta)\big]\,, \tag{3.5b}$$

with A_1, B_1, A_2, B_2 being complex numbers. For each value of the mass m there are two eigenvectors $\chi_m^{(r)}$, $r = 1, 2$, of eq. (3.3), which are normalized as follows,

$$\Big(\chi_m^{(r)}, \chi_{m'}^{(s)}\Big) = \delta^{rs}\delta(m - m')\,. \tag{3.6}$$

The normalization factor is determined by the asymptotics of the modes at large ζ. Making use of the asymptotics of the expressions (3.5) one translates (3.6) into the relation between the coefficients A, B,

$$\frac{2k}{m}\Big(\big(A_{1m}^{(r)}\big)^* A_{1m}^{(s)} + \big(B_{1m}^{(r)}\big)^* B_{1m}^{(s)}\Big) + \frac{2m}{k}\Big(\big(A_{2m}^{(r)}\big)^* A_{2m}^{(s)} + \big(B_{2m}^{(r)}\big)^* B_{2m}^{(s)}\Big) = \delta^{rs}\,. \tag{3.7}$$

Let us turn to the boundary equations (3.2). In terms of the coefficients A, B they take the form,

$$\begin{aligned} A_1\left(J_1\left(\frac{m}{k}\right) - \frac{\lambda}{m}J_2\left(\frac{m}{k}\right)\right) + B_1\left(N_1\left(\frac{m}{k}\right) - \frac{\lambda}{m}N_2\left(\frac{m}{k}\right)\right) - \\ - A_2\frac{\mu}{k}J_1\left(\frac{m}{k}\right) - B_2\frac{\mu}{k}N_1\left(\frac{m}{k}\right) = 0\,, \end{aligned} \tag{3.8a}$$

$$\begin{aligned} -A_1\frac{\mu}{m}J_2\left(\frac{m}{k}\right) - B_1\frac{\mu}{m}N_2\left(\frac{m}{k}\right) + A_2\left(\frac{m}{k}J_0\left(\frac{m}{k}\right) - \frac{\nu}{k}J_1\left(\frac{m}{k}\right)\right) + \\ + B_2\left(\frac{m}{k}N_0\left(\frac{m}{k}\right) - \frac{\nu}{k}N_1\left(\frac{m}{k}\right)\right) = 0\,. \end{aligned} \tag{3.8b}$$

It is convenient to choose the first eigenvector of the operator (3.3) to satisfy the relation

$$\lambda\chi_1^{(1)} + \mu\hat{\chi}_2^{(1)}\big|_{z=0} = 0\,.$$

Then eqs. (3.8) yield

$$A_1^{(1)} = C^{(1)}N_1\left(\frac{m}{k}\right), \qquad\qquad B_1^{(1)} = -C^{(1)}J_1\left(\frac{m}{k}\right),$$

JHEP09(2005)082

$$A_2^{(1)} = -C^{(1)}\frac{\lambda k}{\mu m}N_0\left(\frac{m}{k}\right), \qquad B_2^{(1)} = C^{(1)}\frac{\lambda k}{\mu m}J_0\left(\frac{m}{k}\right),$$

where the normalization constant $C^{(1)}$ is fixed by eq. (3.7). At $m \ll k$ it has the form,

$$C^{(1)} = \pi\left(\frac{m}{2k}\right)^{3/2},$$

implying the following amplitude of the graviton field on the brane

$$\chi_1^{(1)}\big|_{z=0} = \left(\frac{m}{2k}\right)^{1/2}.$$

These modes are analogous to the continuum graviton spectrum in the RS case. They are completely delocalized and provide corrections to the four-dimensional Einstein gravity only at short distances $r \sim 1/k$.

The orthogonal mode is fixed by eq. (3.7). In the general case the corresponding expressions are rather complicated. A considerable simplification occurs, if one assumes the hierarchy between the parameters, $\lambda \ll \mu \ll \nu \ll k \sim M_5$, where $M_5 = (16\pi G_5)^{-1/3}$ is the five-dimensional Planck mass. Since the physics at large distances is governed by light modes we are interested in the region of masses $m^2 \ll \lambda k$. One obtains,

$$A_1^{(2)} = \frac{2\mu}{\nu}\left(\frac{m}{2k}\right)^{3/2}\left(\ln\frac{m}{2k} + \boldsymbol{C}\right), \qquad B_1^{(2)} = \frac{\pi\mu}{\nu}\left(\frac{m}{2k}\right)^{3/2}\frac{1}{1-\frac{2\lambda}{\nu}\ln\frac{m}{k}},$$

$$A_2^{(2)} = -\frac{1}{2}\left(\frac{2k}{m}\right)^{1/2}, \qquad B_2^{(2)} = -\frac{\pi\lambda}{2\nu}\left(\frac{2k}{m}\right)^{1/2}\frac{1}{1-\frac{2\lambda}{\nu}\ln\frac{m}{k}},$$

where $\boldsymbol{C}$ is the Euler constant. An important feature of this mode is the divergence of the amplitudes of the fields on the brane in the small mass limit,

$$\chi_1^{(2)}\big|_{z=0} = -\frac{\mu}{\nu}\left(\frac{2k}{m}\right)^{1/2}\frac{1}{1-\frac{2\lambda}{\nu}\ln\frac{m}{k}},$$

$$\hat{\chi}_2^{(2)}\big|_{z=0} = \frac{\lambda}{\nu}\left(\frac{2k}{m}\right)^{1/2}\frac{1}{1-\frac{2\lambda}{\nu}\ln\frac{m}{k}}.$$

Thus, the modes $\chi_m^{(2)}$ give dominant contribution to the Green's function of the operator (3.3) at large distances on the brane. Their collection comprises the quasilocalized four-dimensional graviton.

To make the last statement more precise, let us consider production of gravitational waves by an external periodic transverse traceless source on the brane,

$$T_{ij}^{\rm ext}(x,z) = \delta(z)\mathrm{e}^{-i\omega t}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\mathrm{e}^{i\mathbf{p}\mathbf{x}}d_{ij}^{(\alpha)}(\mathbf{p})T_\alpha^{\rm ext}(\mathbf{p}).$$

The resulting gravitational field is given by the convolution of the source with the Green's function

$$G(\mathbf{x}-\mathbf{x}';\omega) = -16\pi G_5\int d\tau\; G_{11}(x,x';z=z'=0)\mathrm{e}^{-i\omega\tau}, \tag{3.9}$$

JHEP09(2005)082

where $\tau = t - t'$, and $G_{11}(x, x'; z, z')$ is the upper left element of the retarded Green's function of the operator (3.3),

$$G(x, x'; z, z') = \int_0^\infty dm \sum_{s=1,2} \begin{pmatrix} \chi_{1m}^{(s)}(z)\chi_{1m}^{(s)}(z') & \chi_{1m}^{(s)}(z)\hat{\chi}_{2m}^{(s)}(z') \\ \hat{\chi}_{2m}^{(s)}(z)\chi_{1m}^{(s)}(z') & \hat{\chi}_{2m}^{(s)}(z)\hat{\chi}_{2m}^{(s)}(z') \end{pmatrix} \int \frac{d^4p}{(2\pi)^4} \frac{\mathrm{e}^{-ip(x-x')}}{m^2 - p^2 - i\epsilon p_0} \,. \tag{3.10}$$

Substituting eq. (3.10) into (3.9) one obtains,

$$G(\mathbf{x} - \mathbf{x}'; \omega) = -\frac{4G_5}{r} \int_0^\infty dm \sum_{s=1,2} \left(\chi_{1m}^{(s)}(0)\right)^2 \mathrm{e}^{ip_\omega r} \,, \tag{3.11}$$

where $r = |\mathbf{x} - \mathbf{x}'|$, $p_\omega = \sqrt{\omega^2 - m^2}$ when $m < \omega$ and $p_\omega = i\sqrt{m^2 - \omega^2}$ when $m > \omega$. We see that the gravitational field on the brane has the form of a superposition of massive four-dimensional modes. Only modes with $m < \omega$ are actually radiated, the other ones exponentially fall off from the source. Thus, as long as we are interested in the gravitational waves, we can integrate in (3.11) only up to $m = \omega$. This means, in particular, that at $\omega \ll k$ one can neglect the contribution of the modes $\chi_m^{(1)}$ in Eq. (3.11). Let us introduce

$$m_c = k\mathrm{e}^{-\nu/2\lambda} \,, \tag{3.12}$$

and study the regime $m_c \ll \omega \ll k$. The Green's function (3.11) takes the form

$$G(\mathbf{x} - \mathbf{x}'; \omega) = -\frac{4G_N}{r}\, \mathrm{e}^{i\omega r} \cdot \frac{2\lambda}{\nu} \int_0^\omega \frac{dm}{m} \frac{\mathrm{e}^{-i\frac{rm^2}{2\omega}}}{\left(1 - \frac{2\lambda}{\nu} \ln \frac{m}{k}\right)^2} \,, \tag{3.13}$$

where $G_N = G_5 k$ is the four-dimensional Newton constant. At $r \ll \omega/m_c^2$ the integral in eq. (3.13) is saturated by $m \sim m_c$ and we obtain the usual four-dimensional expression for the gravitational wave. We stress that this wave is a superposition of massive modes with masses of order m_c. In the opposite limit $r \gg \omega/m_c^2$ the integral is damped by the rapidly oscillating exponent, so one obtains

$$G(\mathbf{x} - \mathbf{x}'; \omega) = -\frac{4G_N}{r} \frac{\nu}{\lambda \ln \frac{k^2 r}{2\omega}}\, \mathrm{e}^{i\omega r} \,.$$

The gravitational wave gradually dissipates into the fifth dimension.

A few comments are in order. First, the graviton mass scale m_c turns out to be independent of the parameters $\varkappa_1$, $\varkappa_2$ in the action (2.1):

$$\frac{\nu}{2\lambda} = \frac{1}{16\pi G_5 v^2} \,.$$

This is not very surprising. Indeed, from the very beginning we have been interested in the modes with masses much smaller than the energy scale associated with $\varkappa_1$ and $\varkappa_2$. Then, the potential term for vector fields is effectively frozen at zero and its parameters do not affect dynamics. Rather, the condition that the potential must be zero, generates mixing between gravitons and vectors which leads to quasilocalization of graviton. The strength of mixing at low energies depends only on v, and so does the graviton mass.

JHEP09(2005)082

Second, very small values of m_c are generated without fine-tuning. For example, taking k, M_5 as large as the Planck mass and $v \approx (M_5/5)^{3/2}$ one obtains m_c as small as $(10^{28}\text{cm})^{-1}$, that is the inverse of the present horizon size of the Universe.

Finally, though the Lorentz symmetry is broken in the background (2.2), the Green's function (3.10) for the tensor modes has Lorentz-invariant form.[1] The place where the Lorentz-breaking is essential is that the very notion of the tensor modes refers to the preferred reference frame.

4. Vector modes

The gauge conditions (2.3), (2.6) result in the following equations for the vector perturbations,

$$p_0\psi_1 + \mathbf{p}^2\psi_2 = 0\,, \tag{4.1a}$$

$$p_0\psi_3 + \mathbf{p}^2\psi_4 + \mathbf{p}^2\psi_5 = 0\,, \tag{4.1b}$$

where we again omitted the polarization index α. From Eqs. (2.4), (2.5) one obtains the bulk equations for the vector modes,

$$\frac{1}{2}\psi_I'' - 2k^2\psi_I + \frac{p^2}{2u^2}\psi_I = 0\,, \qquad I = 1,2\,, \tag{4.2a}$$

$$-\psi_I'' + 2k\psi_I' - \frac{p^2}{u^2}\psi_I = 0\,, \qquad I = 3,4,5\,, \qquad z > 0\,, \tag{4.2b}$$

The junction conditions on the brane read:

$$\psi_1' + 2k\psi_1 = 0\,, \tag{4.3a}$$

$$\psi_2' + 2k\psi_2 = \lambda\left(\psi_2 + \frac{2\psi_4}{v} + \frac{i\psi_6}{v}\right)\,, \tag{4.3b}$$

$$\psi_3' = \psi_5' = 0\,, \tag{4.3c}$$

$$\psi_4' = \varkappa^2 v^3\left(\psi_2 + \frac{2\psi_4}{v} + \frac{i\psi_6}{v}\right)\,. \tag{4.3d}$$

Equations (4.3a), (4.3b) together with the gauge fixing condition (4.1a) imply

$$\left(\psi_2 + \frac{2\psi_4}{v}\right)\Big|_{z=0} + \frac{i\psi_6}{v} = 0\,. \tag{4.4}$$

Thus, the system (4.3) reduces to a set of homogeneous boundary conditions for the vector part of the gravitational perturbations and for the free vectors in the RS background. In other words, mixing between gravitational and vector fields has no effect in the sector of vector modes. The relations (4.1), (4.4) imply three independent vector modes, parametrized, say, by the functions $\psi_2(z)$, $\psi_3(z)$, $\psi_4(z)$.

[1] This allows to interpret the additional enhancement of the distance $r \sim \omega/m_c^2$, at which the dissipation of the gravitational waves sets in, by the factor ω/m_c as compared to m_c^{-1}, as a relativistic effect, see ref. [2].

JHEP09(2005)082

Let us show the absence of physical zero modes in the vector sector. Assume that there is a normalizable mode with $p^2 = 0$. Then, from eqs. (4.2) we find that $\psi_2 = C_2 \mathrm{e}^{-2k|z|}$, and that ψ_3, ψ_4 do not depend on z. The latter fact means that the corresponding vector a^a_μ is independent of z, see eq. (2.14e). Consequently the vector field strength $F^a_{\mu\nu}$ is also independent of z, and, for the mode to be normalizable, $F^a_{\mu\nu}$ must be zero. This implies that the vector field a^a_μ of the zero mode is pure gauge in the bulk. From eqs. (2.14b), (2.14c) we find that the metric of the zero mode is also pure gauge,

$$h_{\mu\nu}(x,z) = (\xi_{\mu,\nu} + \xi_{\nu,\mu})\mathrm{e}^{-2k|z|}\,, \qquad \xi_\mu = -iC_{2\alpha} e^{(\alpha)}_\mu \mathrm{e}^{-ipx}\,.$$

Thus, in the bulk, this mode can be removed by residual gauge transformation left after fixing the transverse-traceless gauge (2.3), (2.6). The only degree of freedom we are left with is the vector component ψ_6 of the field β^a, see eq. (2.14f). This field resides on the brane and describes explicit breaking of the $[\mathrm{U}(1)]^3$ gauge symmetry by the brane action, cf. eq. (2.12). But ψ_6 vanishes due to eq. (4.4). Hence, all components of the mode vanish: there is no normalizable zero mode at all. One concludes that there is only the continuum spectrum in the vector sector with $p^2 \geq 0$. The corresponding modes are completely delocalized and have the same form as in the pure RS case.

It is worth mentioning that the fact, that at $p^2 = 0$ the mode ψ_6 vanishes due to field equations and boundary conditions, could be dangerous, if this mode became propagating in a non-trivial background. As we show in appendix B, this is not the case, which implies also that ψ_6 remains non-dynamical to higher orders in classical perturbation theory.

5. Scalar sector

We now turn to the scalar sector. Having in mind the issue of vDVZ discontinuity, we will consider the gravitational field produced on the brane by an external energy-momentum tensor $T^{\mathrm{ext}}_{\mu\nu}$ localized on the brane. We do not restrict our analysis to static sources and consider external energy-momentum tensor depending both on time and space. Passing to the 4-dimensional Fourier representation we decompose it as follows,

$$T^{\mathrm{ext}}_{00} = t_1\,, \qquad T^{\mathrm{ext}}_{0i} = p_i t_2 + \cdots\,, \qquad T^{\mathrm{ext}}_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2}\right) t_3 + p_i p_j t_4 + \cdots\,,$$

where we wrote down explicitly only the scalar components, which are of interest to us. The energy-momentum conservation implies

$$p_0 t_1 + \mathbf{p}^2 t_2 = 0\,, \qquad p_0 t_2 + \mathbf{p}^2 t_4 = 0\,.$$

Below we will consider two gauge-invariant scalar potentials, $\bar\phi_3$ and $\bar\Phi \equiv \bar\phi_1 + 2p_0\bar\phi_2 + p_0^2\bar\phi_4$, of the metric produced by the external source on the brane. The quantities with the bar are defined via decomposition of the metric on the brane in accordance with (2.14a)–(2.14c). The potentials $\bar\phi_3$, $\bar\Phi$ are related to the standard Bardeen variables Φ, Ψ (in the notations of ref. [26]) as follows, $\bar\phi_3 = 2\Psi$, $\bar\Phi = 2\Phi + \frac{2p_0^2}{\mathbf{p}^2}\Psi$. Using the relation (2.11) between the

JHEP09(2005)082

metric induced on the brane and the bulk metric we obtain,

$$\bar{\phi}_3 = \phi_3\big|_{z=0} + 2k\varepsilon\,, \tag{5.1}$$

$$\bar{\Phi} = (\phi_1 + 2p_0\phi_2 + p_0^2\phi_4)\Big|_{z=0} + \frac{2kp^2}{\mathbf{p}^2}\varepsilon\,. \tag{5.2}$$

Our purpose is to compute the potentials $\bar{\phi}_3$ and $\bar{\Phi}$ and to compare them to the expressions obtained in the usual four-dimensional Einstein gravity.

In the bulk, the fields obey the following equations, see eqs. (2.4), (2.5),

$$\frac{1}{2}\phi_I'' - 2k^2\phi_I + \frac{p^2}{2u^2}\phi_I = 0\,, \qquad I = 1,2,3,4\,, \tag{5.3a}$$

$$-\phi_I'' + 2k\phi_I' - \frac{p^2}{u^2}\phi_I = 0\,, \qquad I = 5,6,7\,, \qquad z > 0\,, \tag{5.3b}$$

The gauge conditions (2.3), (2.6) take the form,

$$\phi_1 - 2\phi_3 - \mathbf{p}^2\phi_4 = 0\,, \tag{5.4a}$$

$$p_0\phi_1 + \mathbf{p}^2\phi_2 = 0\,, \tag{5.4b}$$

$$p_0\phi_2 + \mathbf{p}^2\phi_4 = 0\,, \tag{5.4c}$$

$$p_0\phi_5 + \mathbf{p}^2\phi_7 = 0\,. \tag{5.4d}$$

As described in appendix A, the junction conditions in the presence of the external energy-momentum tensor on the brane can be cast into the following form,

$$\phi_1' + 2k\phi_1 = 2\mathbf{p}^2\varepsilon + 8\pi G_5 t_1\,, \tag{5.5a}$$

$$\phi_2' + 2k\phi_2 = -2p_0\varepsilon + 8\pi G_5 t_2\,, \tag{5.5b}$$

$$\phi_3' + 2k\phi_3 = -p^2\varepsilon - 4\pi G_5\frac{p^2}{\mathbf{p}^2}t_1\,, \tag{5.5c}$$

$$\phi_4' + 2k\phi_4 = \frac{2p_0^2}{\mathbf{p}^2}\varepsilon + 8\pi G_5 t_4\,, \tag{5.5d}$$

$$\phi_5' = 0\,, \tag{5.5e}$$

$$\phi_6' = -\frac{3\nu_1}{2\lambda_1}vp^2\varepsilon - 2\pi G_5\frac{\nu_1}{\lambda_1}v(-t_1 + 2t_3 + \mathbf{p}^2 t_4)\,, \tag{5.5f}$$

$$\phi_7' = 0\,, \tag{5.5g}$$

$$-p^2\varepsilon = \lambda_1\rho\left(\phi_3 + \frac{2}{v}\phi_6 + 2k\varepsilon\right) + \frac{4\pi G_5}{3}(-t_1 + 2t_3 + \mathbf{p}^2 t_4)\,, \tag{5.5h}$$

where

$$\nu_1 = 2\varkappa_1^2 v^2\,, \qquad \rho = \frac{\lambda_1 + \lambda_2}{3\lambda_1 + 2\lambda_2}\,.$$

These equations should be supplemented by Eq. (A.2a) in appendix A, which determines the longitudinal component ϕ_8 in terms of the other fields. Note that eqs. (5.5c), (5.5f), (5.5h) form a closed system. Once the solution of this subsystem is found, the other equations are solved in a straightforward manner.

JHEP09(2005)082

Before proceeding with the calculation of the gravitational field produced by the source, let us make sure that the scalar sector does not contain instabilities. So, one temporarily sets the external source equal to zero in eqs. (5.5). First, we show that there is no tachyonic mode, which would correspond to negative or complex p^2. For such a mode, let us introduce w, such that $k^2w^2 = -p^2$ and $\mathrm{Re}\, w > 0$. Then, for a normalizable mode, one would have

$$\phi_3 = U_3 K_2(w\mathrm{e}^{k|z|})\,, \qquad \phi_6 = U_6 w\mathrm{e}^{k|z|} K_1(w\mathrm{e}^{k|z|})\,,$$

where K_1, K_2 are modified Bessel functions. From eqs. (5.5c), (5.5f) we obtain

$$U_3 = -\frac{kw\varepsilon}{K_1(w)}\,, \qquad U_6 = -\frac{3\nu_1 vk\varepsilon}{2\lambda_1 K_0(w)}\,.$$

Inserting these expressions into eq. (5.5h) one obtains the following relation,

$$w + \frac{\lambda_1\rho}{k}\left[\frac{K_0(w)}{K_1(w)} + \frac{3\nu_1}{\lambda_1}\frac{K_1(w)}{K_0(w)}\right] = 0\,. \tag{5.6}$$

Let us show that the real part of the expression in square brackets is positive in the right half-plane $\mathrm{Re}\, w > 0$. Indeed, on the boundary of this half-plane one has,

$$\mathrm{Re}\left[\frac{K_1(w)}{K_0(w)}\right]\Bigg|_{w=iy} = \frac{2}{\pi|y|(J_0^2(y)+N_0^2(y))} > 0\,,$$
$$\mathrm{Re}\left[\frac{K_1(w)}{K_0(w)}\right]\Bigg|_{|w|\to\infty,\,-\pi/2\le \mathrm{Arg}\, w\le\pi/2} = 1 > 0\,.$$

Hence, $\mathrm{Re}[\frac{K_1(w)}{K_0(w)}]$, being a harmonic function, is positive everywhere inside the half-plane $\mathrm{Re}\, w > 0$. This ensures the positivity of the real part of the inverse function, $\mathrm{Re}[\frac{K_0(w)}{K_1(w)}] > 0$, and thus of the whole expression in square brackets in eq. (5.6). One concludes that eq. (5.6) has no solutions in the right half-plane, implying the absence of tachyonic modes in the scalar sector.

Second, let us demonstrate that the model is also free of ghosts. The ghost mode, if any, must be localized on the brane. Indeed, the normalization of the modes of continuum spectrum is determined entirely by the bulk action, which by itself is free of ghosts. It is straightforward to see that the modes corresponding to strictly positive p^2 belong to continuum part of the spectrum. The only dangerous eigenvalue is $p^2 = 0$. But the corresponding mode is unphysical. Indeed, for this mode $\phi_2 = U_2\mathrm{e}^{-2k|z|}$ and from eq. (5.5b) one obtains $\varepsilon = 0$. Substituting $\mathbf{p}^2 = p_0^2$ into the gauge fixing conditions (5.4b), (5.4c) one finds $\phi_1 = -p_0 U_2\mathrm{e}^{-2k|z|}$, $\phi_4 = -\frac{U_2}{p_0}\mathrm{e}^{-2k|z|}$. Then, eq. (5.4a) yields $\phi_3 = 0$. It follows from eq. (5.3b) that the scalar component ϕ_7 of the vector fields does not depend on z. From eq. (5.4d) we find $\phi_5 = -p_0\phi_7$. Finally, from eq. (5.5h) we obtain $\phi_6 = 0$. The surviving mode is pure gauge in the bulk, see eqs. (2.14),

$$h_{\mu\nu} = (\xi_{\mu,\nu} + \xi_{\nu,\mu})\mathrm{e}^{-2k|z|}\,, \qquad \xi_\mu = \frac{p_\mu}{2ip_0}U_2\mathrm{e}^{-ipx}\,,$$
$$a^a_\mu = \partial_\mu\left(-ip_a\phi_7\mathrm{e}^{-ipx}\right).$$

JHEP09(2005)082

The bulk fields can be removed by a residual gauge transformation, leaving the scalar component ϕ_8 (see eq. (2.14f)) of the fields β^a on the brane. But the latter is zero according to the field equation (A.2a). Thus, all the components of the zero mode vanish. There is only continuum spectrum of modes with $p^2 > 0$ in the scalar sector.

We show in appendix B that the mode ϕ_8 remains non-dynamical in a non-trivial background as well. Hence, no new propagating degrees of freedom appear both in non-trivial backgrounds and in higher orders of classical perturbation theory.

Having established the absence of instabilities, we now return to the field produced by the source. Our strategy is to express the functions $\phi_3(z)$, $\phi_6(z)$ in terms of the external source and the brane bending ε using the bulk equations with the boundary conditions (5.5c), (5.5f), and to insert the result into eq. (5.5h). This is most easily done by combining the bulk equations with the boundary conditions into the Schrödinger type operators and introducing the Green's functions of these operators. The latter satisfy the following equations,

$$\begin{aligned} -u^2 G_3''(z;p) + 4k^2 u^2 G_3(z;p) - 4k\delta(z) G_3(z;p) - p^2 G_3(z;p) &= \delta(z)\,, \\ -u^2 G_6''(z;p) + 2ku^2 G_6'(z;p) - p^2 G_6(z;p) &= \delta(z)\,. \end{aligned}$$

Imposing the radiation (outgoing wave) boundary conditions at $z \to \pm\infty$ one obtains,

$$G_3(z;p) = -\frac{H_2^{(1)}(p\mathrm{e}^{k|z|}/k)}{2pH_1^{(1)}(p/k)}\,, \tag{5.7}$$

$$G_6(z;p) = -\frac{\mathrm{e}^{k|z|}H_1^{(1)}(p\mathrm{e}^{k|z|}/k)}{2pH_0^{(1)}(p/k)}\,, \tag{5.8}$$

where $p = \sqrt{p^2}$, $\mathrm{Re}\, p \geq 0$. Making use of these Green's functions we find,

$$\phi_3(0) = -\frac{pH_2^{(1)}(p/k)}{H_1^{(1)}(p/k)}\left(\varepsilon + 4\pi G_5 \frac{t_1}{\mathbf{p}^2}\right), \tag{5.9}$$

$$\phi_6(0) = -\frac{H_1^{(1)}(p/k)}{pH_0^{(1)}(p/k)}\left(\frac{3\nu_1}{2\lambda_1} v p^2 \varepsilon + 2\pi G_5 \frac{\nu_1}{\lambda_1} v(-t_1 + 2t_3 + \mathbf{p}^2 t_4)\right). \tag{5.10}$$

Substitution of these expressions into Eq. (5.5h) yields,

$$\begin{aligned} &\varepsilon\left[p^2 + \lambda_1\rho\frac{pH_0^{(1)}(p/k)}{H_1^{(1)}(p/k)} - 3\nu_1\rho\frac{pH_1^{(1)}(p/k)}{H_0^{(1)}(p/k)}\right] - \\ &-\frac{4\pi G_5}{3}T\left[1 - 3\nu_1\rho\frac{H_1^{(1)}(p/k)}{pH_0^{(1)}(p/k)}\right] - 4\pi G_5\lambda_1\rho\frac{pH_2^{(1)}(p/k)}{H_1^{(1)}(p/k)}\frac{t_1}{\mathbf{p}^2} = 0\,, \end{aligned} \tag{5.11}$$

where we have used the relation

$$2k - \frac{pH_2^{(1)}(p/k)}{H_1^{(1)}(p/k)} = \frac{pH_0^{(1)}(p/k)}{H_1^{(1)}(p/k)}$$

and introduced the notation $T = t_1 - 2t_3 - \mathbf{p}^2 t_4$ for the trace of the energy-momentum tensor.

JHEP09(2005)082

Further analysis depends on the value of the parameter $\varkappa_1^2$. Let us first consider the case $\varkappa_1^2 = 0$, which in eq. (2.1) corresponds to the absence of the first term in the potential for vector fields.[2] In this case $\lambda_1 = \nu_1 = 0$, and eq. (5.11) reduces to,

$$\varepsilon = \frac{4\pi G_5}{3p^2} T \,. \tag{5.12}$$

The last expression coincides with the result obtained in the pure RS case, cf. [24]. Substitution of (5.12) into eqs. (5.9), (5.10) leads[3] to vanishing of the scalar part of the vector fields, $\phi_6 = 0$, while the scalar components of the metric are the same as in the pure RS case. Hence, there are only short-distance corrections to the scalar part of the metric in the case $\varkappa_1^2 = 0$. At distances $r \gg 1/k$, i.e. at small momenta $p \ll k$, one arrives at the same expressions as in the four-dimensional Einstein gravity,

$$\bar{\phi}_3 = -\frac{8\pi G_N}{\mathbf{p}^2} t_1 \,, \qquad \bar{\Phi} = \frac{8\pi G_N p^2}{(\mathbf{p}^2)^2} t_1 - \frac{16\pi G_N}{\mathbf{p}^2} t_3 \,, \tag{5.13}$$

where $G_N = G_5 k$ is the four-dimensional Newton constant. Evidently, the gravitational field is free from the vDVZ discontinuity. In particular, the Newton law — the field of a static source — does not get modified at large distances in the case $\varkappa_1^2 = 0$.

Another possibility is that the parameter $\varkappa_1^2$ is large. In this case we are interested in the behavior of the fields at distances $r \gg 1/\nu_1$, i.e., at small values of the momentum, $p \ll k, \nu_1$. We also assume as in section 3 the hierarchy $\lambda_1 \ll \nu_1$. The dominant contributions to the expressions in the square brackets in eq. (5.11) come from the terms proportional to $H_1^{(1)}(p/k)/H_0^{(1)}(p/k)$. It is worth noting that these terms come from the scalar component ϕ_6 of the vector fields in eq. (5.5h). Keeping only the leading contributions at low momenta, one obtains from eq. (5.11),

$$\varepsilon = \frac{4\pi G_5}{3p^2} T + \frac{8\pi G_5}{3\mathbf{p}^2} t_1 \frac{\lambda_1}{\nu_1} \ln \frac{p}{k} \,. \tag{5.14}$$

This expression for the brane bending differs from the formula (5.12) in the pure RS case by the last term, which is small at moderate momenta. However, it becomes important and is logarithmically large relative to the first term in the far infrared. Making use of the expression (5.14) for the brane bending ε and the Green's function (5.7) one determines the rest of the gravitational modes, ϕ_1, ϕ_2, ϕ_4. Finally, these expressions, being inserted into eqs. (5.1), (5.2), yield the scalar components of the metric induced on the brane,

$$\bar{\phi}_3 = -8\pi G_N \frac{t_1}{\mathbf{p}^2} \,, \tag{5.15}$$

$$\bar{\Phi} = \frac{8\pi G_N p^2}{(\mathbf{p}^2)^2} t_1 - \frac{16\pi G_N}{\mathbf{p}^2} t_3 + \frac{16\pi G_N p^2}{(\mathbf{p}^2)^2} t_1 \frac{\lambda_1}{\nu_1} \ln \frac{p}{k} \,. \tag{5.16}$$

The expression (5.15) coincides with the result of the usual four-dimensional Einstein gravity. Thus, the potential ϕ_3 does not get modified at large distances at all. As to the second

[2] We do not know whether vanishing of the parameter $\varkappa_1^2$ can be ensured by any symmetry requirement.

[3] Note that though $\lambda_1 = \nu_1 = 0$, the ratio $\frac{\nu_1}{\lambda_1} = \frac{1}{8\pi G_5 v^2}$ in (5.10) is finite.

JHEP09(2005)082

potential, the first two terms in eq. (5.16) also coincide with the result of conventional gravity. The last term provides the long-distance correction, which becomes important at the distance

$$r_c = \frac{1}{k} e^{\nu_1/2\lambda_1} = \frac{1}{k} e^{1/16\pi G_5 v^2} .$$

This distance coincides with $1/m_c$, where m_c is the graviton mass scale (3.12).

To make the physical consequences of the formula (5.16) more clear, let us calculate the gravitational field produced by a point-like static source. We take $T_{00} = M\delta(\mathbf{x})$, $T_{0i} = T_{ij} = 0$. Setting $t_1 = M$, $t_3 = 0$, $p^2 = -\mathbf{p}^2$ in eq. (5.16) and performing Fourier transform, one obtains,

$$\bar{\Phi}(r) = -\frac{2G_N M}{r}\left(1 - \frac{2\lambda_1}{\nu_1}\ln kr\right). \tag{5.17}$$

while for the other gauge invariant potential we have,

$$\bar{\phi}_3(r) = -\frac{2G_N M}{r}. \tag{5.18}$$

These expressions are valid up to corrections at small distances $r \sim 1/k, 1/\nu_1$. The potential (5.17) is the analog of the Newton potential in our model, it is responsible for gravitational interaction between nonrelativistic massive objects. Remarkably, a contribution to $\bar{\Phi}(r)$ appears, which grows logarithmically with the distance as compared to the standard expression $(-2G_N M/r)$. Even more strikingly, the sign of this contribution is opposite to that of the standard expression. As a result, the expression (5.17) describes gradual weakening of gravitational attraction with the distance; at large distances, $r > 1/m_c$, attraction gets replaced by repulsion. Thus, our model provides an example of a ghost-free theory with antigravity at ultra-large distances.

At first sight it seems surprising that in the model with quasilocalized gravitons we obtain a contribution to the Newton potential which grows at large distances, as compared to the standard four-dimensional expression. Following the conventional line of reasoning one could infer that, as gravitons dissipate at large distances into the fifth dimension, the gravitational interaction should become weaker at large distances, as compared to the four-dimensional case. However, this line of reasoning is incorrect. In theories with Lorentz symmetry breaking, the potential produced by an external source is not directly related to the spectrum of propagating degrees of freedom. This point is illustrated by four-dimensional Lorentz violating massive electrodynamics, considered in refs. [27]. In that case the electric potential of static sources falls off as $1/r$ in spite of the fact that all propagating modes are massive.

The origin of the logarithmically enhanced antigravity in our model can be understood as follows. A point mass gives rise to perturbations of the vector fields which interact with matter via mixing with the metric. The gravitational field is produced by the total energy-momentum tensor composed of $T^{\rm ext}_{\mu\nu}$ and the energy-momentum tensor $T^V_{\mu\nu}$ of the vectors, given by eq. (2.9). The latter tensor falls off slowly from the localized external source, and dominates at large distances. Our results indicate that, insofar as the Newton potential is concerned, the vector fields mimic the effect of negative energy. This is possible because $T^V_{\mu\nu}$ violates the weak energy condition.

JHEP09(2005)082

The gravitational potentials (5.15), (5.16) are free from the vDVZ discontinuity. Indeed, in the limit of vanishing graviton mass scale m_c, which corresponds to $\lambda_1/\nu_1 \to 0$, one recovers the usual four dimensional expressions (5.13). However, from the phenomenological point of view the difference between the two gravitational potentials (5.17) and (5.18) results in a severe phenomenological constraint on the value of the parameter λ_1/ν_1. Measurements of the light deflection by the gravitational field of the Sun [28] require $\lambda_1/\nu_1 \lesssim 10^{-5}$. This value is not unnaturally small: it corresponds to $v \sim (0.027 M_5)^{3/2}$. However, it pushes the range $1/m_c$, where the antigravity sets in, far beyond the present horizon size of the Universe. The mass scale m_c in this case is very small and it is not clear whether it can significantly affect the physics within the present horizon. We discuss possible ways of avoiding this phenomenological constraint in the concluding section.

6. Discussion

In this paper we presented a Lorentz-violating brane-world model, where gravitons are not completely localized, but are rather quasilocalized on the brane. In other words, the four-dimensional graviton is a collection of KK modes from the continuum spectrum. The characteristic mass m_c of these modes is exponentially small when expressed in terms of the parameters of the lagrangian. We demonstrated that the model is free from tachyonic and ghost-like instabilities. We calculated the metrics produced by an external energy-momentum tensor on the brane, and found that the model does not suffer from the vDVZ discontinuity.

The key observation behind the model is that while the gravitational perturbations in the Randall-Sundrum setup contain a zero mode localized on the brane, the bulk vector fields are completely delocalized. Thus, mixing between the vectors and the would-be zero gravitational mode forces the latter to dissipate into the fifth dimension. This mixing between tensors and vectors becomes possible when the Lorentz symmetry is spontaneously broken by non-zero VEVs of the vector fields. These considerations are rather generic. We believe them to be applicable to a broad class of generalizations of the Randal-Sundrum model with spontaneous Lorentz symmetry breaking by bulk fields. In particular, the vector fields can be replaced by form-fields of higher degrees, which are also completely delocalized in the Randall-Sundrum background.

For general form of the potential term for the vector fields on the brane the model exhibits antigravity at ultra-large distances. Namely, the structure of the Newton potential is such that gravitational attraction between nonrelativistic massive objects gradually weakens as the distance increases, and gets replaced by repulsion at $r > 1/m_c$. While this property is theoretically appealing, it puts severe phenomenological constraints on the model. The reason is that the second Bardeen potential, characterizing spatial part of the metric of a static source, does not get modified at large distances and thus differs from the Newton potential. Observations of light deflection by the Sun constrain the relative difference between the two gravitational potentials at the level of 10^{-4}. This translates into the constraint $m_c \lesssim M_{Pl} \exp(-10^5)$. This value is negligible compared to the present Hubble

JHEP09(2005)082

parameter of the Universe. It is doubtful whether the model with so tiny graviton mass scale can lead to any interesting phenomenology, in particular, to accelerating expansion of the Universe at the present epoch.

One possibility to avoid this phenomenological problem is to fine-tune the parameter $\varkappa_1$ of the vector potential in the action (2.1) to zero. Then, both Bardeen potentials have at large distances the same form as in the four-dimensional linearized Einstein gravity, while graviton is still quasilocalized and gravity waves dissipate into extra dimensions. The graviton mass scale m_c in this case can be comparable to the Hubble parameter or even larger. A drawback of this approach is fine-tuning of $\varkappa_1$. We are not aware whether vanishing of this parameter can be imposed by any symmetry.

Another way out is a generalization of the model considered in this paper. Let us sketch a particular example. One introduces a scalar field with dilaton-like coupling to the vector fields on the brane. Namely, one replaces the combination $\bar{g}^{\mu\nu}A^a_\mu A^b_\nu$ in the brane action in eq. (2.1) by $\bar{g}^{\mu\nu}\mathrm{e}^{\alpha\varphi}A^a_\mu A^b_\nu$. The modified model allows for the Lorentz-violating background (2.2) with $\varphi = 0$. The dilaton does not affect tensor and vector sectors of the linearized perturbation above this background, so the modified model also incorporates quasilocalized gravitons. On the other hand, relative difference between Bardeen potentials depends on the dilaton coupling, and is essentially proportional to $1/\alpha^2$ when α is large. Thus, the constraint from light deflection is satisfied once $\alpha \gtrsim 100$. We will report more on phenomenology of our model and its generalizations elsewhere.

Another important issue is the quantum structure of the theory. Of special interest is the scale where the strong coupling at the quantum level sets in. Our analysis shows that in our model there are no modes with anomalously large propagators, which usually pull the strong coupling scale to unacceptably low energies. On the other hand, a subtle point in our model is the existence of constraints (4.4), (A.2a). They lead to the presence of non-propagating modes in the theory. Let us briefly discuss the effects associated with virtual exchange of these modes. For simplicity we switch off gravity by taking the limit $G_5 \to 0$ (cf. appendix B). Then, the modes of interest are purely longitudinal modes localized on the brane, $\bar{a}^a_\mu = \partial_\mu \beta^a$. Propagators for these modes come from the potential on the brane and are inversely proportional to the square of the spatial mometum,

$$\langle \beta^a \beta^b \rangle \propto \frac{1}{\varkappa^2 v^2} \cdot \frac{\delta^{ab}}{\mathbf{p}^2} \,. \tag{6.1}$$

Evidently, they do not contain any large parameters. But, in principle, these modes could provide dangerous contributions to the Green functions: naively, the propagator (6.1) attached to a vertex with time derivative of β^a produces a factor of the form $p_0/|\mathbf{p}|$. Such factors can become large in some kinematic regimes. The question whether this does or does not affect the strong coupling scale in our model is left for future investigations.

Last, but not least, is the question what kind of late-time cosmological evolution is obtained in the model (2.1) and its generalizations; in particular, whether they can account for the cosmic acceleration at the present epoch.

JHEP09(2005)082

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A. Junction conditions in the scalar sector

In this appendix we consider the junction conditions for the scalar modes in the presence of an external energy-momentum tensor on the brane. The external source should be added to the r.h.s. of eqs. (2.13) according to eq. (2.7). As a result one obtains the following junction conditions on the brane,

$$\phi_1' + 2k\phi_1 - 2p_0^2\varepsilon = \frac{\lambda_1}{3}\left(2\phi_3 + \mathbf{p}^2\phi_4 + \frac{4}{v}\phi_6 + \frac{2\mathbf{p}^2}{v}\phi_7 - \frac{\mathbf{p}^2}{k}\varepsilon + 6k\varepsilon + \frac{2i\mathbf{p}^2}{v}\phi_8\right) + \\ + 8\pi G_5\left(\frac{2}{3}t_1 + \frac{2}{3}t_3 + \frac{\mathbf{p}^2}{3}t_4\right), \tag{A.1a}$$

$$\phi_2' + 2k\phi_2 + 2p_0\varepsilon = 8\pi G_5 t_2\,, \tag{A.1b}$$

$$\phi_3' + 2k\phi_3 = \lambda\left(\frac{1}{3}\phi_3 - \frac{\mathbf{p}^2}{3}\phi_4 + \frac{2}{3v}\phi_6 - \frac{2\mathbf{p}^2}{3v}\phi_7 + \frac{\mathbf{p}^2}{3k}\varepsilon - \frac{2i\mathbf{p}^2}{3v}\phi_8\right), \tag{A.1c}$$

$$\phi_4' + 2k\phi_4 - 2\varepsilon = \lambda\left(-\frac{2}{3\mathbf{p}^2}\phi_3 + \frac{2}{3}\phi_4 - \frac{4}{3\mathbf{p}^2 v}\phi_6 + \frac{4}{3v}\phi_7 - \frac{2}{3k}\varepsilon + \frac{4i}{3v}\phi_8\right) + \\ + 8\pi G_5\left(\frac{1}{3\mathbf{p}^2}t_1 - \frac{2}{3\mathbf{p}^2}t_3 + \frac{2}{3}t_4\right), \tag{A.1d}$$

$$\phi_5' = 0\,, \tag{A.1e}$$

$$\phi_6' = \varkappa_1^2 v^3\left(\phi_3 + \frac{2}{v}\phi_6 + 2k\varepsilon\right) + \\ + \varkappa_2^2 v^3\left(\frac{1}{3}\phi_3 - \frac{\mathbf{p}^2}{3}\phi_4 + \frac{2}{3v}\phi_6 - \frac{2\mathbf{p}^2}{3v}\phi_7 + \frac{\mathbf{p}^2}{3k}\varepsilon - \frac{2i\mathbf{p}^2}{3v}\phi_8\right), \tag{A.1f}$$

$$\phi_7' = \varkappa_1^2 v^3\left(\phi_4 + \frac{2}{v}\phi_7 - \frac{1}{k}\varepsilon + \frac{2k}{\mathbf{p}^2}\varepsilon + \frac{2i}{v}\phi_8\right) + \\ + \varkappa_2^2 v^3\left(-\frac{2}{3\mathbf{p}^2}\phi_3 + \frac{2}{3}\phi_4 - \frac{4}{3\mathbf{p}^2 v}\phi_6 + \frac{4}{3v}\phi_7 - \frac{2}{3k}\varepsilon + \frac{4i}{3v}\phi_8\right). \tag{A.1g}$$

Combining these equations with the gauge fixing conditions (5.4), we obtain

$$\lambda_1\left\{\left(-\mathbf{p}^2\phi_4 - \frac{2\mathbf{p}^2}{v}\phi_7\right)\Big|_{z=0} + \frac{\mathbf{p}^2}{k}\varepsilon - 2k\varepsilon - \frac{2i\mathbf{p}^2}{v}\phi_8\right\} + \\ + \lambda_2\left\{\left(\frac{2}{3}\phi_3 - \frac{2\mathbf{p}^2}{3}\phi_4 + \frac{4}{3v}\phi_6 - \frac{4\mathbf{p}^2}{3v}\phi_7\right)\Big|_{z=0} + \frac{2\mathbf{p}^2}{3k}\varepsilon - \frac{4i\mathbf{p}^2}{3v}\phi_8\right\} = 0\,, \tag{A.2a}$$

JHEP09(2005)082

$$-2p^2\varepsilon = \frac{\lambda_1}{3}\left\{\left(2\phi_3 + \mathbf{p}^2\phi_4 + \frac{4}{v}\phi_6 + \frac{2\mathbf{p}^2}{v}\phi_7\right)\Big|_{z=0} - \frac{\mathbf{p}^2}{k}\varepsilon + 6k\varepsilon + \frac{2i\mathbf{p}^2}{v}\phi_8\right\} + \\ + 8\pi G_5\left(-\frac{1}{3}t_1 + \frac{2}{3}t_3 + \frac{\mathbf{p}^2}{3}t_4\right). \qquad \text{(A.2b)}$$

The first of these equations plays the role similar to that of eq. (4.4) in the vector sector: it determines the field ϕ_8 in terms of the other fields. Substitution of eqs. (A.2) back into eqs. (A.1) yields the system (5.5) considered in the main text.

B. Absence of ghosts above a non-trivial background

We saw in the main text that the modes ϕ_8 and $\psi_{6\alpha}$, accounting for the explicit $[\mathrm{U}(1)]^3$ gauge symmetry breaking on the brane, vanish to the linear order in perturbation theory above the background (2.2). These vanishing modes are generically dangerous. In higher orders of classical perturbation theory (i.e. above non-trivial backgrounds) kinetic terms for these modes may arise, rendering them propagating. Depending on the sign of kinetic term some of these modes may become ghosts. To study this issue we investigate the ultraviolet behavior of the perturbations of the vector fields above a non-trivial vector background close to the static background $A_i^a = v\delta_i^a$. For the sake of simplicity we neglect gravity perturbations. This can be done consistently by taking the limit $G_5 \to 0$; we believe that switching on gravity does not spoil our results. The non-trivial background is chosen to be locally space-like $A_\mu^a = (0, \mathbf{A}^a)$. In this appendix we show that the equations of motion for the linear perturbations above the non-trivial vector background imply vanishing of the dangerous modes, in complete analogy to the situation above the static background (2.2). This demonstrates the ultraviolet stability of the model (2.1) to higher orders in classical perturbation theory.

Let us consider the brane part of the quadratic action for the fluctuations a_μ^a of the vector fields above the non-trivial vector background,

$$S^{(2)} = \int d^4x \Big[-\left(\varkappa_1^2 + \varkappa_2^2\right)\left(A^{\mu\ b}A_\mu^c a^{\nu\ b}a_\nu^c + A^{\mu\ b}a_\mu^c a^{\nu\ b}A_\nu^c + A^{\mu\ b}a_\mu^c a^{\nu\ c}A_\nu^b\right) - \\ - \varkappa_1^2 v^2 \cdot a^{\nu\ b}a_\nu^b + \frac{\varkappa_2^2}{3}\left(A^{\mu\ b}A_\mu^b a^{\nu\ c}a_\nu^c + 2A^{\mu\ b}a_\mu^b A^{\nu\ c}a_\nu^c\right)\Big],$$

where summation over internal indices $b, c = 1, \ldots, 3$ is assumed, and the metric on the brane is taken to be flat. The bulk part of the action is irrelevant for us as we are interested in longitudinal modes $a_\mu^a = \partial_\mu \beta^a$ which are pure gauge in the bulk.

Let us consider equations for the spatial components of the fluctuations, $a_i^a = \partial_i \beta^a$. It is convenient to represent them in the matrix form introducing 3×3 matrices $\hat{a} = a_i^a$ and $\hat{A} = A_i^a$. By a suitable transformation, the background matrix $\hat{A}$ can be made symmetric, $\hat{A} = \hat{A}^T$. Then the equation of motion for the fluctuations reads,

$$-\left(\varkappa_1^2 + \varkappa_2^2\right)\left(\hat{A}^2\hat{a} + \hat{A}\hat{a}^T\hat{A} + \hat{a}\hat{A}^2\right) + \varkappa_1^2 v^2 \cdot \hat{a} + \frac{\varkappa_2^2}{3}\left(2\mathrm{Tr}[\hat{a}\hat{A}]\cdot\hat{A} + \mathrm{Tr}[\hat{A}^2]\cdot\hat{a}\right) = 0\,. \quad \text{(B.1)}$$

The symmetric part of the fluctuations, $s_i^a = \partial_i\beta^a + \partial_a\beta^i$, obeys the homogeneous equation,

$$\mathcal{A}[\hat{s}] = 0\,,$$

JHEP09(2005)082

where the linear operator $\mathcal{A}$ can be read from eq. (B.1). For the static background (2.2b) the operator $\mathcal{A}$ is non-degenerate. Thus, it is non-degenerate for non-trivial backgrounds, which are close enough to (2.2b). One concludes that the matrix $\hat{s}$ vanishes,

$$\partial_i \beta^b + \partial_b \beta^i = 0\,. \tag{B.2}$$

Differentiating eq. (B.2) twice yields the Laplace equation for the divergence, $\Delta \partial_a \beta^a = 0$. Imposing vanishing boundary conditions at spatial infinity we obtain $\partial_a \beta^a = 0$. Then, differentiating eq. (B.2) once, we find that all the components β^a vanish.

Thus the longitudinal modes ϕ_8 and $\psi_{6\alpha}$ do not become propagating in non-trivial background. This result implies the absence of ghosts and/or tachyons in the model (2.1) to higher orders in classical perturbation theory.

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